

DIOCESE OF TOLEDO

MATHEMATICS COURSE OF STUDY

HIGH SCHOOL



Catholic Youth and School Services
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**MATHEMATICS COURSE OF STUDY
REVISION COMMITTEE
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DIOCESE OF TOLEDO PHILOSOPHY

“The duty of human perfection, like the whole universe, has been renewed, recast, supernaturalized, in the Kingdom of God. It is a truly Christian duty to grow... and to make one’s talents bear fruit...It is a part of the essentially Catholic vision to look upon the world as maturing--not only in each individual or in each nation, but in the whole human race.”

(Teilhard de Chardin, The Divine Milieu)

The schools of the Catholic Diocese of Toledo assist parents in preparing their youngsters to assume their Christian vocation. The schools enable youngsters to perfect and grow in the knowledge, skills, values and attitudes to which they are called by Jesus Christ. This vocation begins and grows as each member hears the message of the Gospel, seeks to achieve a personal relationship with Jesus Christ and shares in a commitment of love and service of God and others in order to transform self and society.

Christian education in the Toledo diocesan schools is intended to make students become people of faith who can experience--inside and outside the school setting--learning and living in the light of this faith commitment. Students are instructed in human knowledge and skills in order to best relate human culture to God’s plan for his evolving creation. Religious education, i.e., instruction in truths and development of values, is of primary concern. This religious education serves as the basis by which students can integrate their experiences of learning and living at each stage of their development.

This integration thrives in a thoroughly Christian atmosphere where faculties and staffs share and demonstrate in their professional and private lives this same commitment to personal perfection and growth in Jesus Christ.

Toledo diocesan schools enable students to extend their personal faith commitment through prayer and by serving others. Together with faculty and staff, students participate in liturgical activities which foster community. Students explore ways to meet the challenges of tensions and conflicts which occur in community, especially in peacemaking and the achievement of justice. Gospel values impel students to special concern for all who suffer any disadvantage. Students are enabled to commit themselves to the public interest by developing the skills and talents needed to contribute to the life of the nation.

This experience of integrating learning and living a commitment of faith is a reason for hope. It is the duty of the schools of the diocese of Toledo to continually explore and rekindle hope for the future in the light of the present reality of the universe. Engaging our members--and the community-at-large--in a search for growth and perfection is our never-ending obligation. Our ultimate goal is union with Jesus Christ, “the way, the truth and the life.”

DIOCESE OF TOLEDO

PHILOSOPHY OF THE MATHEMATICS PROGRAM

The ability to reason is a God-given gift, and development of that gift is central to the mission of the Catholic school. Mathematical literacy equips students to develop problem-solving strategies for discovering creative and humane solutions to the challenges offered by our society. The diocesan mathematics program is designed to be developmentally appropriate, recognizing student differences in learning styles through the use of manipulatives and cooperative learning at all levels. Mathematics contributes to the development of students by enriching their lives and enabling them to acquire the knowledge and skills needed to meet the opportunities and the demands of an increasingly technologically-based society.

The mathematics program is a dynamic process, periodically reviewed and updated, incorporating the new and changing needs of individuals and society.

OVERVIEW OF THE MATHEMATICS COURSE OF STUDY

This Mathematics Course of Study follows the same format as recommended by the National Council of Teachers of Mathematics and by the Ohio Mathematics Content Standards. Both the N.C.T.M and the state standards organize material into various math “standards” that are present throughout all grade levels, K-12.

The Mathematics Course of Study for the Diocese of Toledo is presented in the same way. Therefore teachers will find material at each grade level organized by standards. It should be noted that the material is not organized sequentially. The material does not necessarily need to be learned by the students in the order presented. A further note, the “Mathematical Processes Standard” is shown at the beginning of each grade level, but also should be imbedded in all the other standards as well.

Standards:

Mathematical Processes Standard

Number, Number Sense, and Operations Standard

Measurement Standard

Geometry and Spatial Sense Standard

Patterns, Functions, and Algebra Standard

Data Analysis and Probability Standard

MATHEMATICAL PROCESSES STANDARD:

Through the study of mathematics, students will discover the complexities and wonders of creation and experience God. In addition, mathematics will provide students with tools with which to build a better world. They live out their faith based on social justice principles, using mathematical processes and knowledge to solve problems. Therefore, it is essential for students in Catholic elementary and high schools to focus not only on the content presented, but also on the process.

Problem Solving:

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof:

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication:

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections:

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

NUMBER, NUMBER SENSE AND OPERATIONS STANDARD:
Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

1. Recognize the significance of the history of mathematics as it connects to every content area:
 - Investigate the development of numbers,
 - Research and appreciate the contributions of mathematicians,
 - Explore and discuss classic problems; e.g., Koenigburg Bridge, handshake problem, double money.

2. Understands numbers, ways of representing numbers, relationships among numbers and number systems:
 - Compare, order and determine equivalent representation of real numbers,
 - Compare, contrast and connect the properties of numbers and number systems (complex numbers, imaginary numbers, real numbers, rational numbers, integers, and irrational numbers),
 - Understand vectors and matrices as systems that have some properties of the real-number system.

3. Understand meanings of operations and how they relate one another:
 - Recognize the connections between the operations of addition, subtraction, multiplication, division, computing powers and roots and how they are related,
 - Understand the effects operations on the size of numbers,
 - Understand permutations and combinations as counting techniques.

4. Compute fluently and make reasonable estimates:
 - Develop fluency in operations with complex numbers, real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more complicated cases,
 - Decide when a problem situation is best solved using mental arithmetic, paper and pencil, calculator, or a computer,
 - Analyze and evaluate the reasonableness of numerical computations and their results,
 - Develop a sense of appropriateness of estimation and calculator-generated answers,
 - Apply factorials to solve problems.

MEASUREMENT STANDARD:

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

1. Understand measurable attributes of objects and the units, systems, and processes of measurement; e.g., make decisions about units and scales that are appropriate for problem situations involving measurements.
2. Apply appropriate techniques, tools, and formulas to determine measurements:
 - Solve problems using measurements and analyze precision, accuracy, and approximate error,
 - Understand and use formulas for the area, surface area, and volume of two- and three-dimensional geometric figures,
 - Apply informal concepts of successive approximation, upper and lower bounds, and limits in measurement situations,
 - Use estimation and unit analysis to check measurement computations,
 - Develop measuring techniques to apply for measuring both regular and irregular objects recognizing the need for flexibility.

GEOMETRY AND SPATIAL SENSE STANDARD:

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships:
 - Use correct vocabulary and symbolic notation to demonstrate an understanding of geometric concepts,
 - Analyze and determine properties of two- and three- dimensional objects
 - Explore relationships; e.g., congruence and similarity among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them,
 - Establish validity of geometric conjectures using inductive and deductive reasoning, prove theorems, and critique arguments made by others,
 - Use trigonometric ratio relationships to solve mathematical and real-world problems; e.g., Law of Sines and Law of Cosines.

2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems:

- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations,
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates,
- Use algebraic means to justify geometric ideas in coordinate proofs; e.g., parallel and perpendicular slopes, the Pythagorean Theorem, distance and midpoint formulas.

3. Apply transformations and use symmetry to analyze mathematical situations:

- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices,
- Use various representations (including technology) to help understand the effects of simple transformations and their compositions and communicate these clearly to others.

4. Construct:

- Right triangles, equilateral triangles, parallelograms, trapezoids, rectangles, rhombuses, squares, and kites using compass and straightedge or dynamic geometry software,
- Congruent or similar figures using tools such as compass, straightedge, protractor, or dynamic geometry software.

PATTERNS, FUNCTIONS AND ALGEBRA STANDARD:

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities, Students analyze, model and solve problems using various representations such as tables, graphs and equations.

1. Understand patterns, relations, and functions:

- Generalize patterns using explicitly and recursively defined functions,
- Use and interpret various representations of relations and functions and convert flexibly among them,
- Perform and analyze transformations; e.g., combining, composing, and inverting on commonly used functions, using technology on more complicated functions,
- Analyze functions of one variable by investigating rates of change, intercepts, zeroes, asymptotes, and local and global behavior,

- Use appropriate terminology and notation to define functions and their properties; e.g., domain, range, composition, and inverse function,
- Recognize and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions,
- Understand the basic concept of the inverse of a function including trigonometric functions and the relation between the graph of a function and its inverse,
- Understand the relationship of trigonometric functions derived from the unit circle.

2. Represent and analyze mathematical situations and structures using algebraic symbols:

- Understand the meaning of equivalent forms of expressions, ratios, equations, inequalities, and relations,
- Use symbolic algebra to represent and explain mathematical relationships; e.g., writing expressions, proportions, equations,
- Write equivalent forms of equations, inequalities, and systems of equations,
- Perform operations on polynomial expressions including factoring, simplifying and expanding,
- Perform operations on matrices and use matrices to solve systems of equations,
- Solve equations and inequalities mentally, algebraically, and graphically, included are polynomial, absolute value, logarithmic, exponential, rational, and radical,
- Solve trigonometric equations and verify trigonometric identities,
- Understand inverse trigonometric functions, their domains and ranges,
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

3. Use mathematical models to represent and understand quantitative relationships:

- Identify essential quantitative relationships in a situation and recognize the functions that might model the relationships,
- Use symbolic expressions to model relationships arising from various contexts,
- Understand and interpret graphs and draw inferences from them,
- Draw reasonable conclusions about a situation being modeled and develop and evaluate an argument for the mathematical conjecture.

4. Analyze change in various contents; e.g., approximate and interpret rates of change from graphical and numerical data.

DATA ANALYSIS AND PROBABILITY STANDARD:

Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

1. Formulate questions that can be addressed with data; then collect, organize and display relevant data to answer them:
 - Know the characteristics of well-designed studies; e.g., survey, experimental, observational, by identifying population, by using a variety of sampling techniques and discern which type of inferences can legitimately be drawn from each,
 - Understand the terminology of measurement data, categorical data, statistic, parameter and margin of error,
 - Compute basic statistics (mean, median, mode and range) and determine which measure of central tendency is appropriate,
 - Understand bar graphs, line graphs, pie charts, histograms, frequency tables, box plots, stem-and-leaf plots and scatterplots and use them to display data.

2. Select and use appropriate statistical methods to analyze data:
 - For single (univariate) measurement data, be able to display the distribution, describe its shape and select and calculate summary statistics
 - Recognize how linear transformations; e.g., adding or subtracting a constant, of single measurement data affect the shape, center and spread,
 - For two-variable (bivariate) measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations and correlation coefficients using technological tools,
 - Identify trends in two-variable data and find functions that model the data or transform the data so they can be modeled, including data where at least one variable is categorical.

3. Develop and evaluate inferences and predictions that are based on data:
 - Use simulations to explore experimental probability,
 - Use sampling and recognize its role in statistical claims,
 - Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis and the validity of conclusions,
 - Understand how basic statistical techniques are applied in the workplace.

4. Understand and apply basic concepts of probability:
 - Apply the concepts of sample space and probability distributions and create sample spaces and distributions,

- Design simulations to construct experimental probability distributions,
- Understand the concepts of experimental and theoretical probability,
- Understand how to compute the probability of independent, dependent and compound events.